

How difficult it would be to detect Cosmic Neutrino Background ?

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Hot Big-Bang Cosmology

(concordance model of cosmology)

explains everything we know about the evolution of the Universe since early times with remarkable accuracy.

In particular from the Big-Bang Nucleosynthesis (first few minutes) and from of the Cosmic Microwave Background (~400 ky) it follows that at these epochs relativistic neutrinos of ~3 flavors were present.

$$N_{\nu}^{\text{BBN}} = 3.71^{+0.47}_{-0.45} \text{ (from D, } ^4\text{He) (Steigman 2012)}$$

$$N_{\nu}^{\text{CMB}} = 3.52^{+0.48}_{-0.45} \text{ (Planck collaboration 2013 uses also BAO and } H_0, \text{ when BICEP2 is included } N_{\nu} \sim 4)$$

Neutrinos decouple when the expansion rate exceeds the interaction rate: $\sigma \sim G_F^2 (kT)^2$, $n_{\nu} \sim (kT)^3$, $t_{\nu} = (n_{\nu}\sigma v)^{-1} \sim G_F^{-2} (kT)^{-5}$, $t_{\text{expansion}} \sim G_N^{-1/2} (kT)^{-2}$, $\Rightarrow kT \sim 1 \text{ MeV}$, $t_{\text{decoupling}} \sim 1 \text{ second}$.

Using elementary consideration one can show that

$$n_{\nu}/n_{\gamma} = 3/11, \text{ thus } \sim 112 \text{ neutrinos of each Majorana flavor /cm}^3$$

and $T_{\nu}/T_{\gamma} = (4/11)^{1/3} = 0.71$; $T_{\nu} = 1.94 \text{ K} = 1.67 \times 10^{-4} \text{ eV}$

These are then firm predictions of the Hot Big-Bang Cosmology:

Neutrino number density = 112 neutrinos/cm³ for each flavor, i.e., 56 neutrinos and 56 antineutrinos of each flavor

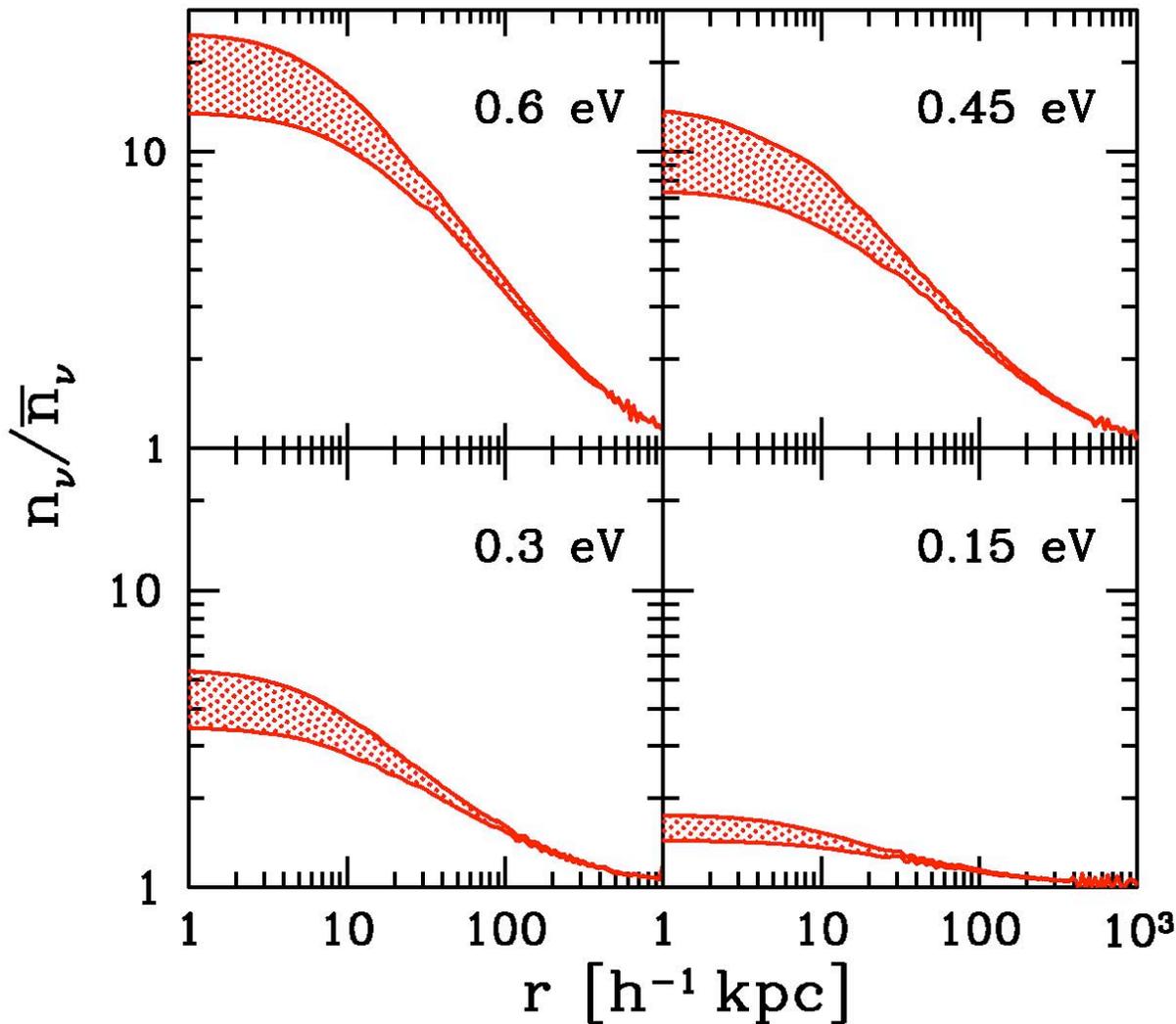
Neutrino temperature = 1.94 K = 1.67x10⁻⁴ eV

If one could confirm (or find deviations) from these predictions, one would test the theory at $t \sim 1$ sec, $T \sim 1$ MeV, and redshift $z \sim 10^{10}$, much earlier and hotter than the tests based on BBN and CMB.

There is, therefore, strong motivation to try to detect these CνB.

Clustering neutrino density enhancement

Massive particles become nonrelativistic when their mass exceeds the temperature of the Universe. From then on they can become bound, i.e., concentrate in structures of various sizes.



Clustering evaluation for the Milky Way (Ringwald & Wong 04)
In fact, for $m \sim 0.1 \text{ eV}$ the overdensity is essentially absent.

Three very different methods of $C\nu B$ detection were proposed:

- 1) Use coherent scattering to detect the drag force of the $C\nu B$ caused by the “neutrino wind”
- 2) Observe the dip in the UHE neutrino flux caused by the resonant annihilation on $C\nu B$ producing Z bosons.
- 3) Charge current reactions of the $C\nu B$ on zero threshold unstable targets.

Use of coherent scattering:

Note that the de Broglie wavelength $\lambda_\nu = h/p_\nu \sim 2.4 \text{ mm}$ (for $p_\nu \sim 3T_\nu$)

A sphere with $d = \lambda_\nu$ contains $\sim 10^{21}$ nucleons. If neutrinos interact coherently with all of them, it should help a lot.

The first ideas, from ~ 1980 , were based on the belief that the effect is linear in G_F . That is, unfortunately, incorrect, only G_F^2 effects are possible. (Cabibbo & Maiani, 82; Langacker, Leveille & Sheiman, 83).

So, as proposed by Shvartsman, Braginski, Gershtein, Zeldovich, and Khlopov, 82 scatter relic neutrinos on spheres with $r = \lambda$; use the virial motion of Earth with respect to the relic neutrinos, $v \sim 300 \text{ km/s}$ and measure the force on such spheres.

However, despite the N_c enhancement the resulting acceleration $a \sim 3 \times 10^{-25} \text{ cm s}^{-2}$, which is many orders of magnitude from the sensitivity of the current Dicke - Eotvos type experiments.

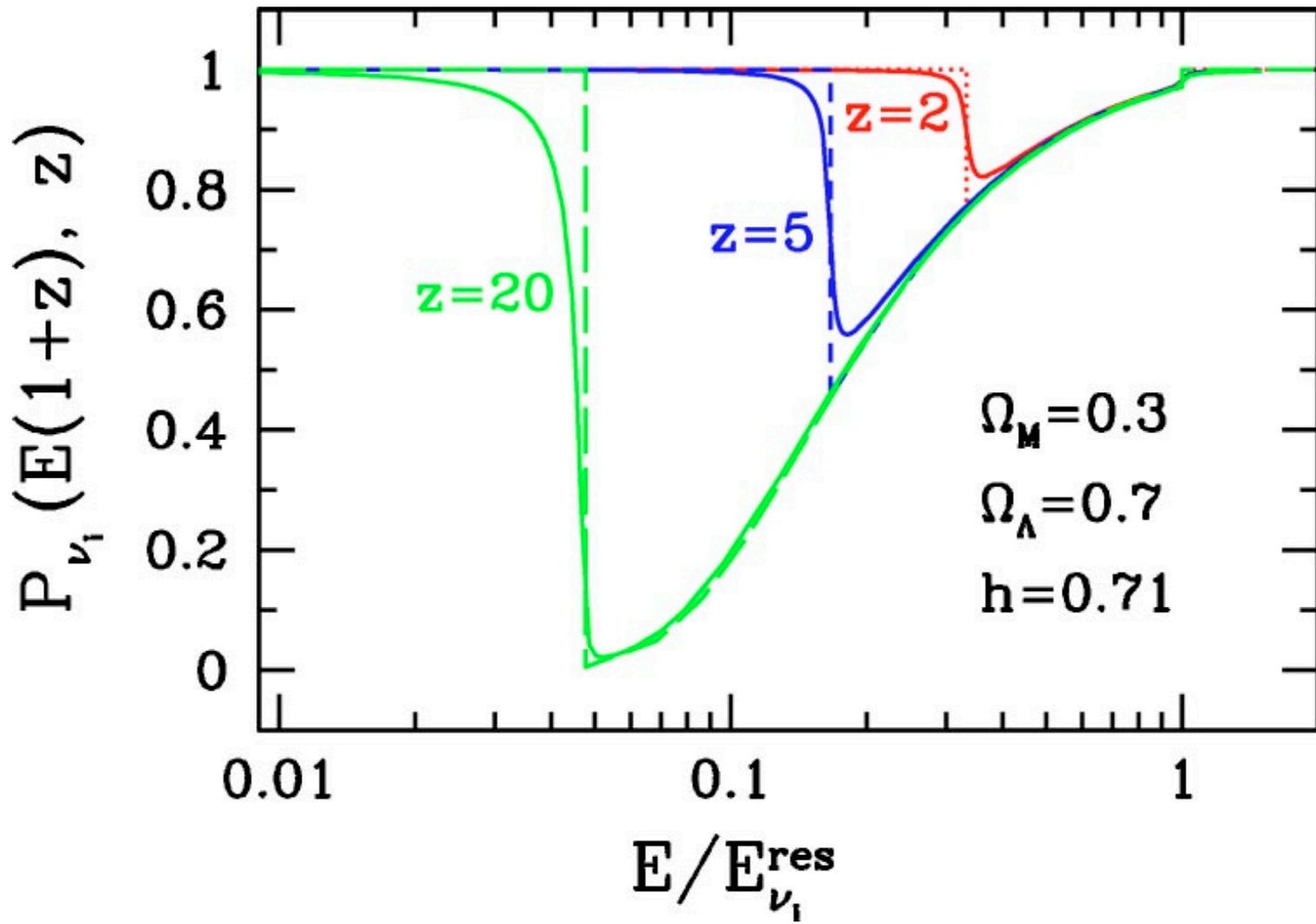
Using resonance absorption of UHE neutrinos on CνB:

The Universe is transparent to neutrinos with the exception of the resonance annihilation into Z-bosons (Weiler 82).

The resonance energy is $E_\nu^{\text{res}} = m_Z^2/2m_\nu = 4.2 \times 10^{22} \text{ eV} (0.1 \text{ eV}/m_\nu)$, and the cross section is $\langle \sigma_{\nu\nu}^{\text{ann}} \rangle = 2\pi\sqrt{2}G_F = 40.4 \text{ nb}$.

When the UHE neutrinos are injected at redshift z with energy E_i , they may be detected at Earth with $E = E_i/(1+z)$. Thus, the ``dip'' in the observed spectrum will be broadened and z dependent.

Clearly, the observable effect will depend on the z and energy distribution, so far unknown, of the UHE neutrino sources.



Survival probability of a cosmic neutrino injected at redshift z with energy E_i , so that at Earth it has energy $E = E_i/(1+z)$, in units of the resonance energy $E_{\nu}^{res} = m_z^2/2m_{\nu}$. Full treatment (full lines) and the narrow width approximation are compared (from Eberle et al, 04)

Since these proposals do not work, by a large margin, let's consider the usual way of detecting neutrinos, by charged current weak interactions.

The problems to solve:

- 1) Can one find an appropriate target?
- 2) How many target atoms can one use in practice?
- 3) What is the cross section, and is the event rate sufficient?
- 4) Can one separate the signal from background?

Each of these items is challenging, but it turns out that the needed technological improvements are *only(??!!)* few orders of magnitude each, so it is worthwhile to consider them in more detail. In fact, a proposed **PTOLEMY** experiment (arXiv 1307.4738) aims at achieving the relic neutrino detection.

First, note that at the sub eV energies the CνB flux dominates over any other neutrino fluxes (reactors, solar) by a large factor.

Second, since the momentum of the CNB $p_\nu \rightarrow 0$, we must consider only exothermic reaction, i.e., reactions on unstable targets.

What is the behavior of the cross section when $p_\nu \rightarrow 0$?

What about the exothermic (hypothetical, there are no free neutrons) $\nu_e + n \rightarrow e^- + p$ with $E_e = M_n - M_p + E_\nu > 0$ even for $E_\nu \rightarrow 0$.

$$\frac{d\sigma}{d\cos\theta} = \frac{\bar{G}^2}{v_\nu} E_e p_e [(f^2 + 3g^2) + (f^2 - g^2)v_e v_\nu \cos\theta]$$

The cross section now contains $1/v_\nu$, which means that the rate, σv_ν , remain finite even when $v_\nu \rightarrow 0$.

(see Weinberg 62, Cocco, Mangano, Messina 07)

Consider now reactions on unstable nuclear targets A_Z



where the allowed β^\pm decay of $A_{Z\pm 1}$ is characterized by the known nuclear matrix element $|M_{nucl}|^2 \approx 6300/ft_{1/2}$.

The cross section in cm^2 for these exothermic reactions is

$$\sigma = \sigma_0 \times \left\langle \frac{c}{v_\nu} E_e p_e F(Z, E_e) \right\rangle \frac{2I' + 1}{2I + 1}$$

with

$$\sigma_0 = \frac{G_F^2 \cos^2 \theta_C m_e^2}{\pi} |M_{nucl}|^2 = \frac{2.64 \times 10^{-41}}{ft_{1/2}}$$

When $v_\nu \rightarrow 0$ the e^\pm energies are monoenergetic $E_e = Q + m_e + m_\nu$ and thus the ν capture signal is separated from the end of the β -decay spectrum by $2m_\nu$.

We can consider now the answer to our first question:

Can one find an appropriate target?

Clearly the unstable A_Z target should have half-life $t_{1/2}$ longer than the duration of the measurement, i.e.,

$t_{1/2} \geq \text{years}$.

It could be manmade, or it could exist in nature. However, natural radioactivity has $t_{1/2} \geq 10^9 \text{ years}$.

The target A_Z should also have minimal possible $ft_{1/2}$ so that the cross section is as large as possible. This means that the superallowed decays, with $ft_{1/2} \sim 1000$ are preferred.

Now, let's consider the second question:

How many target atoms can one use in practice?

When reviewing possible targets, the tritium (^3H) clearly comes to mind. Its half-life $t_{1/2} = 12.3$ years is just right, and $ft_{1/2} = 1143$ is almost as small as the $ft_{1/2}$ for the free neutron decay.

The technology of production is well developed, and using as much as **1 Mcu (2.1×10^{25} tritium atoms)** is very challenging but appears to be technologically possible.

This corresponds to just ~ 100 g of pure tritium.

(Note, however, that the Karlsruhe facility, handling all tritium for the KATRIN experiment, as well as for ITER, is licensed for maximum only 20 g of tritium.)

Now the third question:

What is the cross section, and the event rate?

To estimate the relic neutrino velocity, let's neglect the virial motion and use $v_\nu/c \sim 3T_\nu/m_\nu$, with $T_\nu = 1.9$ K.

With this assumption $\sigma = 1.5 \times 10^{-41} (m_\nu/eV) \text{ cm}^2$

The CNB capture rate per tritium atom is independent of m_ν ,

$$R = \sigma \times v_\nu \times n_\nu \approx 1.8 \times 10^{-32} \times n_\nu / \langle n_\nu \rangle \text{ s}^{-1} \text{ (independent of } v_\nu)$$

And the number of events is

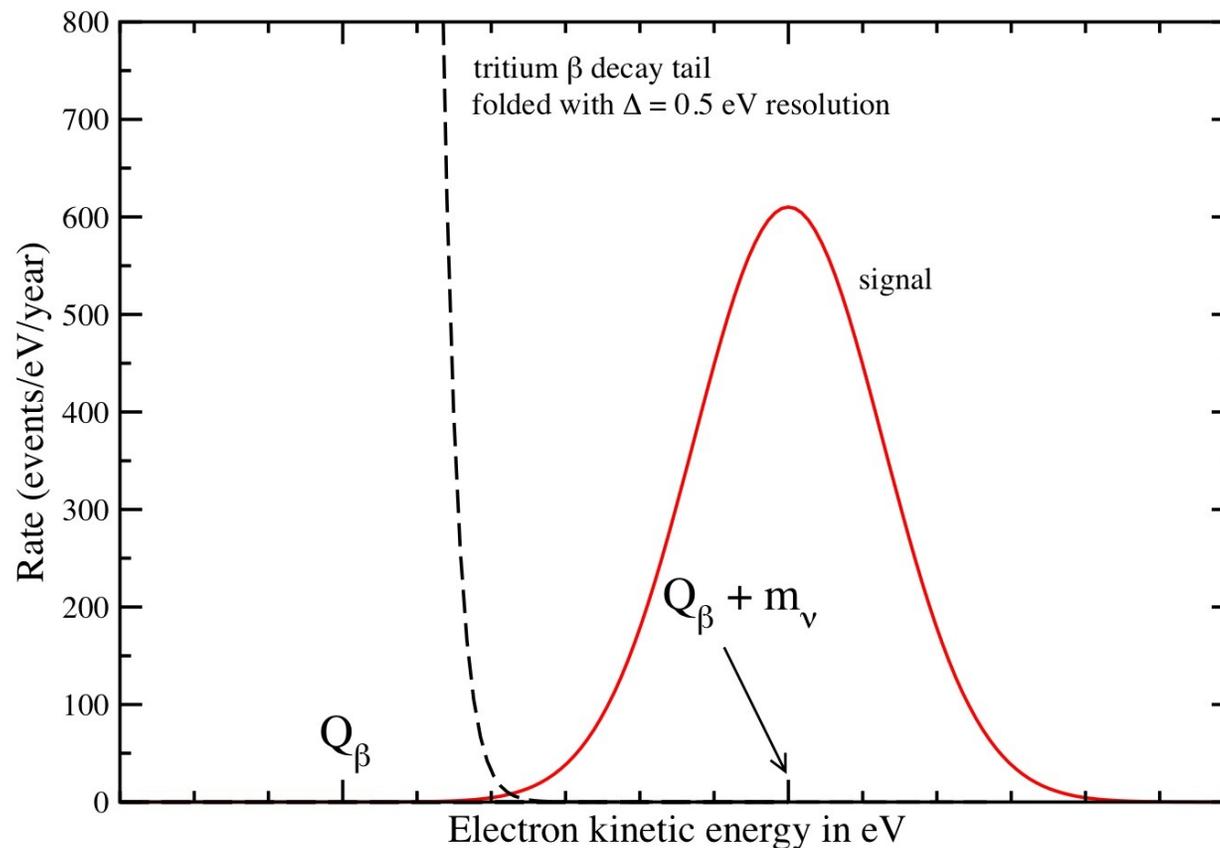
$$N_{\nu \text{ capt}} \approx 830 \text{ yr}^{-1} \text{ Mcu}^{-1} \text{ for } n_\nu / \langle n_\nu \rangle = 100$$

So, the number of events would be reasonably large.

Finally, the last and most difficult question:

Can one separate the signal from background?

There are 3.7×10^{16} tritium β decays/s, and hence emitted electrons distributed over the energy interval $0 - Q_\beta - m_\nu$ and smeared by the detector energy resolution. The fraction of electrons in the energy interval of width Δ just below the endpoint is $\sim (\Delta/Q_\beta)^3$

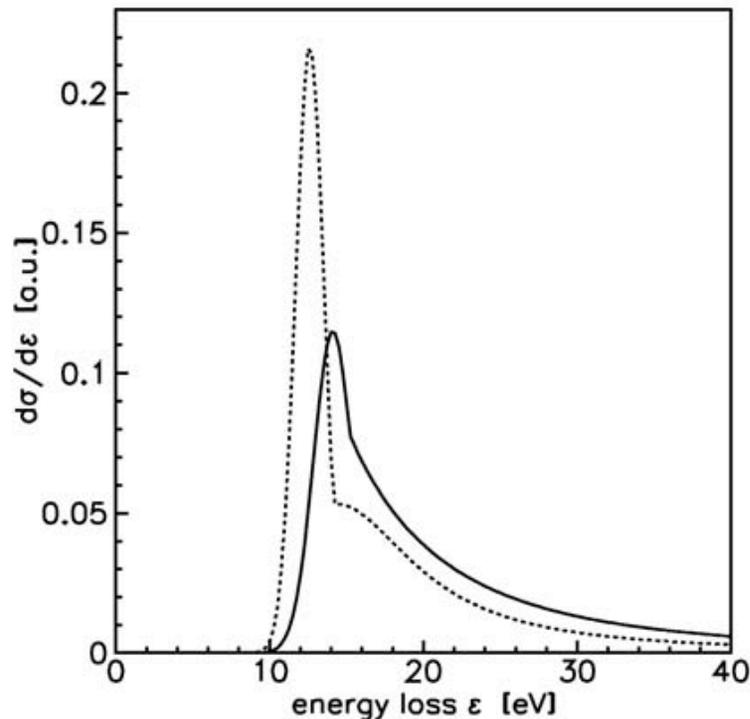


This is for $\Delta = 0.5$ eV
 $m_\nu = 1$ eV and
 $n_\nu / \langle n_\nu \rangle = 50$.

Detailed calculation suggests that in order to achieve signal/background ~ 1 one needs $\Delta \sim m_\nu/2$.

Here are potential killer problems:

- 1) Past and planned experiments use molecular T_2 . The rotational-vibrational states in the final ${}^3\text{HeT}$ molecule are spread over ~ 0.36 eV. That essentially limits the achievable resolution. However, using atomic T would be very difficult.
- 2) Electrons scatter on T_2 with $\sigma = 3 \times 10^{-18} \text{cm}^2$. This limits the source column density and makes much stronger sources impossible. Totally new arrangement would be needed for stronger sources.



Schematic idea of the 'Project 8' of Monreal and Formaggio Phys. Rev. D80, 051301(2009).

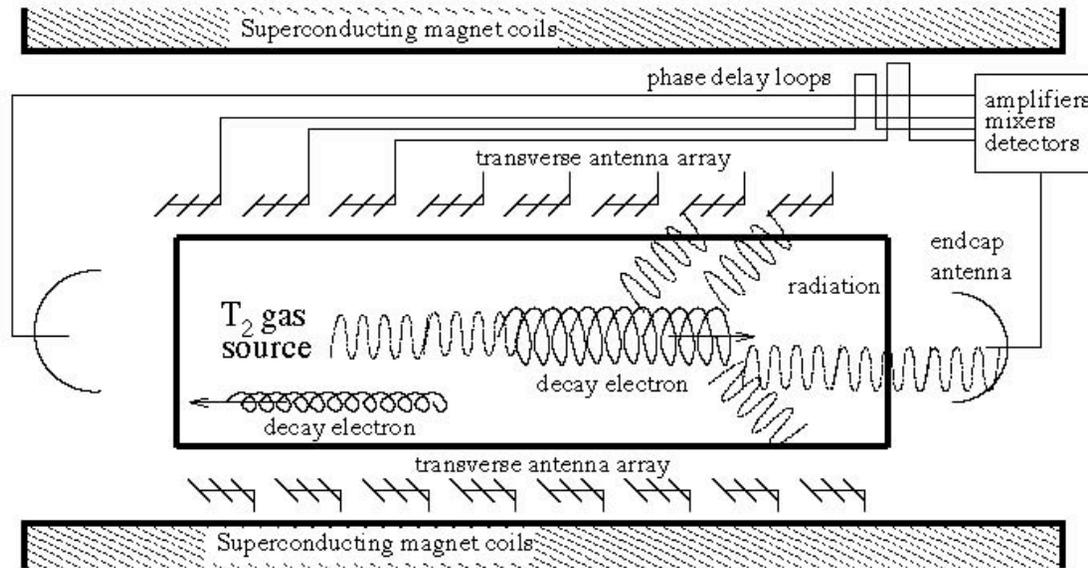


FIG. 1: Schematic of the proposed experiment. A chamber encloses a diffuse gaseous tritium source under a uniform magnetic field. Electrons produced from beta decay undergo cyclotron motion and emit cyclotron radiation, which is detected by an antenna array. See text for more details.

Cyclotron frequency depends on the electron kinetic energy:
 $\omega = qB / (m_e + E)$

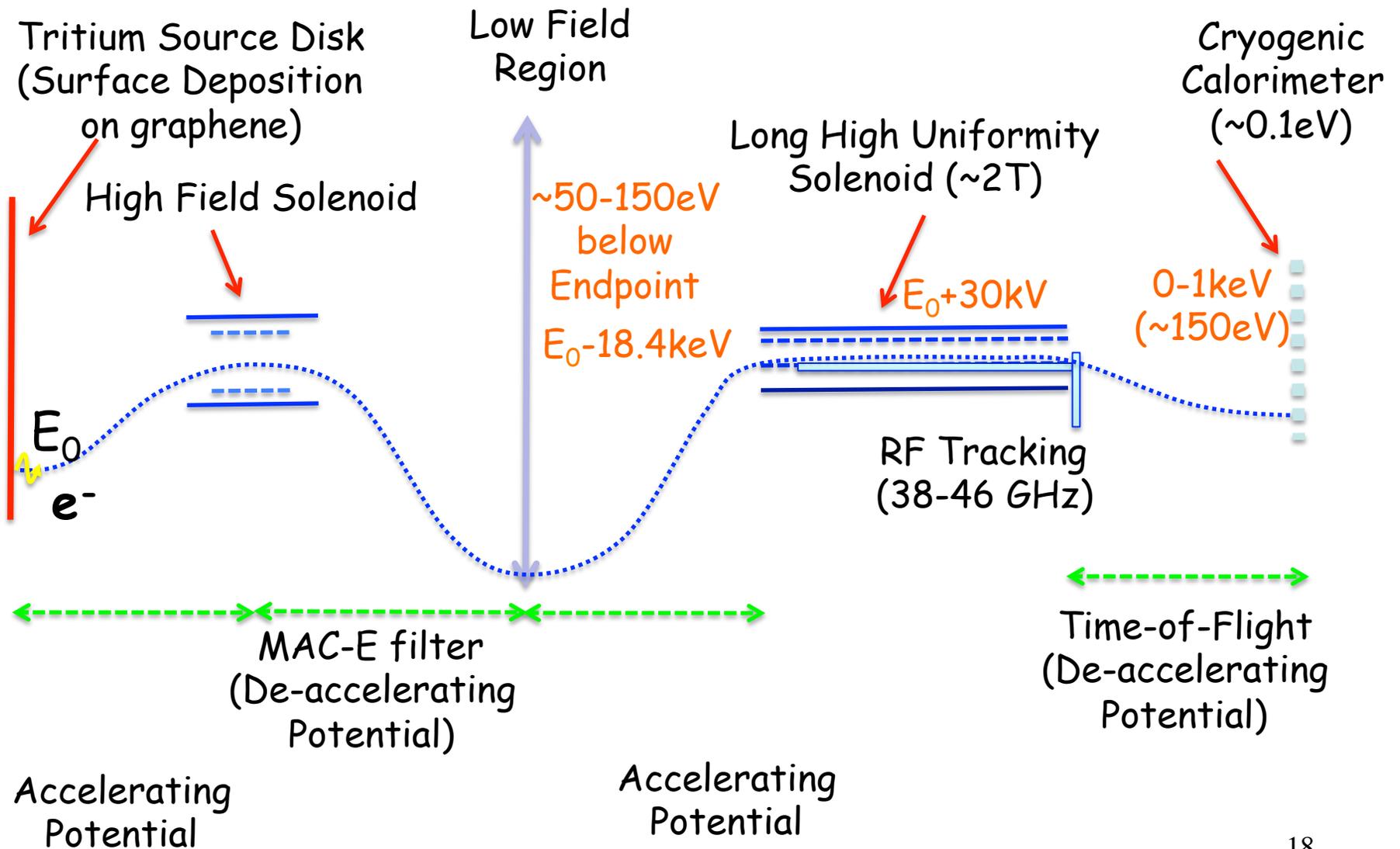
Each electron emits microwaves at frequency ω and total power

$$P(\beta, \theta) = \frac{1}{4\pi\epsilon_0} \times \frac{2q^2\omega^2}{3cx\beta^2 \sin^2\theta} (1 - \beta^2)$$

where β is the electron velocity and θ is the pitch angle

With 100Ci source of atomic tritium the projected sensitivity to neutrino mass of 0.007 eV is estimated.

PTOLEMY: Princeton Tritium Observatory for Light, Early-Universe, Massive-Neutrino Yield



Summary

- 1) We have discussed the challenges of detecting the primordial neutrinos (in particular the ν_e component) using the neutrino capture on radioactive nuclei, with emphasis on tritium as target.
- 2) Among the various technological challenges of such program, the requirement that the detector resolution is better than the neutrino mass by a factor 2 - 3, combined with the use of a very strong source appears to be the most difficult one to achieve. It essentially restricts the applicability of the discussed approach.
- 3) In the next few years a variety of approaches (KATRIN, cosmology & astrophysics, $0\nu\beta\beta$ decay) promise to reach sensitivity to $m_\nu \sim 0.2$ eV or even better. If these approaches will find positive evidence, e.g.. if we can conclude that $m_\nu \geq 0.1$ eV (degenerate neutrino mass scenario) it would be certainly worthwhile, and perhaps even imperative, to pursue the indicated program vigorously.

Spare

Hot Big-Bang Cosmology

(concordance model of cosmology)

explains everything we know about the evolution of the Universe since early times with remarkable accuracy.

In particular, two independent ways of determining the baryon average density (or the ratio of baryons to photons), one from the **Big-Bang Nucleosynthesis** (first few minutes), and the other from analysis of the temperature fluctuations of the **Cosmic Microwave Background** (~400 ky) agree very well.

Both sets of data also agree (with rather large error bars) on the prediction that relativistic neutrinos of ~3 flavors were present at those epochs. These neutrinos have not interacted since that time, thus they should be around us until now. In fact, these neutrinos are expected to be the second (after CMB photons) most abundant particles in the Universe.

$$N_{\nu}^{\text{BBN}} = 3.71^{+0.47}_{-0.45} \text{ (from D, } ^4\text{He) (Steigman 2012)}$$

$$N_{\nu}^{\text{CMB}} = 3.52^{+0.48}_{-0.45} \text{ (Planck collaboration 2013 uses also BAO and } H_0, \text{ when BICEP2 is included } N_{\nu} \sim 4)$$

In the radiation dominated epoch energy density and temperature evolve as

$$\rho = 3c^2/(32\pi G_N) t^{-2}; \quad kT = [45 h^3 c^5 / (32\pi^3 G_N g_s^*)]^{1/4} t^{-1/2},$$
$$kT/\text{MeV} \sim (t/s)^{-1/2}$$

Where $g_s^* = 1 + 7/4 + 3 \times 7/8$ (photons, electrons, 3 neutrino flavors)

Neutrinos decouple when the expansion rate exceeds the interaction rate:

$$\sigma \sim G_F^2 (kT)^2, \quad n_\nu \sim (kT)^3, \quad t_\nu = (n_\nu \sigma v)^{-1} \sim G_F^{-2} (kT)^{-5}$$

$$t_{\text{expansion}} \sim G_N^{-1/2} (kT)^{-2}$$

(t_ν - interval between weak interactions, t_{exp} - characteristic expansion time)

From $t_\nu = t_{\text{exp}} \Rightarrow kT \sim 1 \text{ MeV}$, $t_{\text{decoupling}} \sim 1 \text{ second}$

(detailed calculations give $kT(\nu_e) \sim 2 \text{ MeV}$, $kT(\nu_\mu, \nu_\tau) \sim 3 \text{ MeV}$),

While in equilibrium the number density of each Majorana neutrino flavor is proportional to the photon number density

$$n_\nu/n_\gamma = 3/4 \quad (\text{for relativistic Fermi and Bose gases})$$

At $t \sim 10$ s , e^+ and e^- annihilate increasing n_γ .

That process conserves entropy, $s \sim \rho/T$

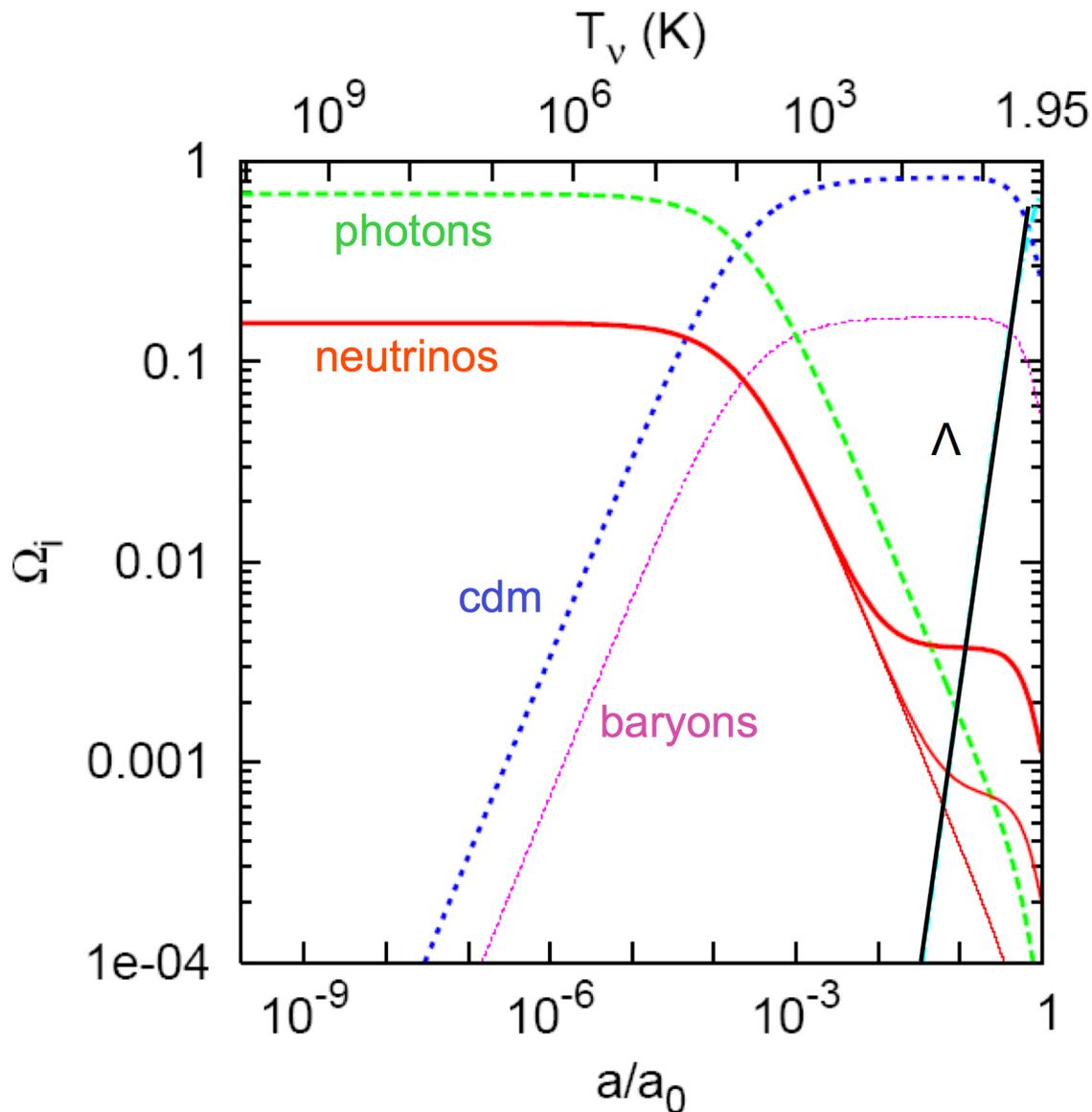
Thus the photon density n_γ increases by the factor $(1 + 2 \times 7/8) = 11/4$

$$n_\nu = (4/11)(3/4) n_\gamma \sim 112 \text{ neutrinos of each Majorana flavor /cm}^3$$

$$\text{and } T_\nu/T_\gamma = (4/11)^{1/3} = 0.71; \quad T_\nu = 1.94 \text{ K} = 1.67 \times 10^{-4} \text{ eV}$$

Neutrinos keep their momentum distribution as that of a relativistic Fermi gas, even when nonrelativistic. However, virial motion in the galactic halo will modify the momentum distribution

$$f_\nu(p, T) = \frac{1}{e^{p/T_\nu} + 1}$$



$$\Omega_i = \rho_i / \rho_c$$

$$\rho_c = 3H^2 / 8\pi G_N$$

$$\rho_c \sim 5 \text{ keV/cm}^{-3}$$

$$\Omega_{\text{tot}} = 1 \text{ is assumed}$$

Background energy densities as a function of temperature (or scale a). Evaluated from $T = 1 \text{ MeV}$ until now with $h_{100} = 0.7$. The neutrino curves are for $m_1 = 0$, $m_2 = 0.009 \text{ eV}$ and $m_3 = 0.05 \text{ eV}$. Massless particles scale like a^{-4} , nonrelativistic particle scale like a^{-3} , and ρ_Λ is time independent.

Clustering neutrino density enhancement

Massive particles become nonrelativistic when their mass exceeds the temperature of the Universe. From then on they can become bound, i.e., concentrate in structures of various sizes. Their densities in these structures can far exceed the average density derived from cosmological measurements and arguments.

component	average ρ (keV/cm ³)	Structure	Enhancement
baryons	0.2	galaxy(disk)	$\sim 5 \times 10^6$
dark matter	1.0	galaxy(halo)	$\sim 3 \times 10^5$
Neutrinos	$112(\Sigma m_\nu/\text{keV})$	clusters	$\sim 1 - 100$

Cosmic background neutrinos can become bound only in structures where their velocity is less than the escape velocity of the structure. For nonrelativistic neutrinos the thermal velocity is $v_{\text{th}}/c = \langle p \rangle/m \sim 3.15 T_\nu/m$.

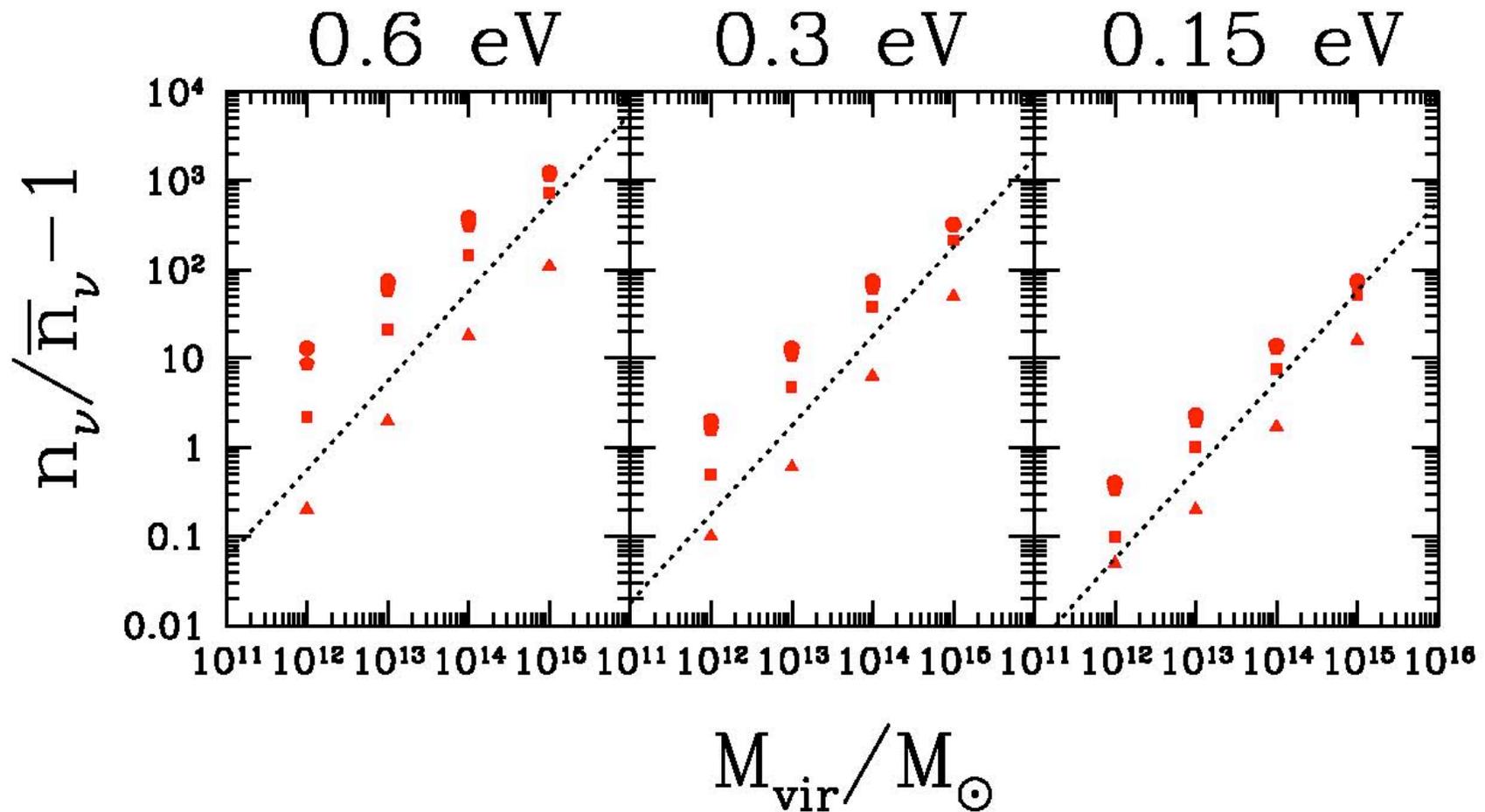
The average velocity of nonrelativistic neutrinos at redshift z is $\langle v_\nu \rangle = 160(1+z)(\text{eV}/m_\nu) \text{ km/s}$. Since galaxies and clusters have velocity dispersion $10^2 - 10^3 \text{ km/s}$ thus sub-eV neutrinos can cluster only at $z \lesssim 2$

Dependence of the overdensity on the mass of the cluster and on the neutrino mass (from Ringwald & Wong, 04, similar to Singh & Ma 04)

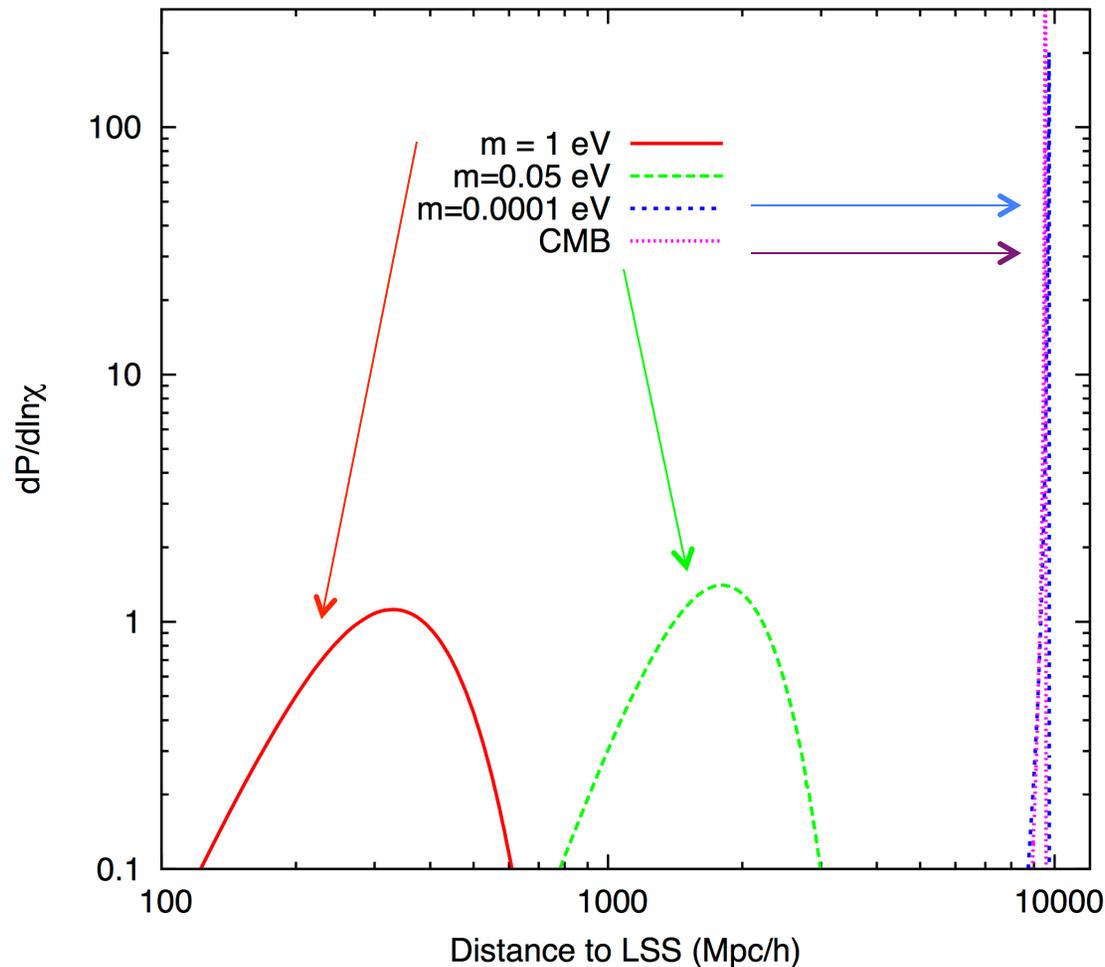
The red symbols indicate different distances from the cluster center,

▲ are for $r = 1 \text{ Mpc}/h$.

For $M_{\text{vir}} = 10^{15} M_{\odot}$, $m_{\nu} > 0.3 \text{ eV}$ our estimate $n_{\nu}/\langle n_{\nu} \rangle = 100$ looks OK



An interesting and counterintuitive consequence of finite nuclear mass, and thus the fact that neutrinos are nonrelativistic now, is the fact that the last scattering surface for them is much closer than for the CMB photons even though they decoupled earlier.



The probability that a neutrino of mass m last scatters at a given comoving distance from us. The large spread is the consequence of the momentum distribution of the neutrinos.

Nevertheless these distances are larger than the size of the largest superclusters.

Can we understand that it is possible to have a considerably larger neutrino capture rate with only ~100g of tritium compared with ~500 ton (fiducial) of scintillator in KamLAND?

Here are the ratios tritium/KamLAND:

Cross section	~100
Number of targets	~ 5×10^{-7}
Flux	~ 10^5
Total	~5